

UNIT-5.

* Vibrations * periodic and aperiodic motion of body

Vibration :- A periodic and regular motion of a body

It is defined as that the shake of object & oscillations of a body in form of waves, that waves is called vibrations.

Free vibrations (a) Natural vibrations.

The elastic vibrations in which there are no friction and external force after the initial release of the body are called as free vibrations (b) natural vibrations.

Damped vibrations :-

When the energy of vibrating such as a gradually dissipated by friction and other resistance, the vibrations are said to be damped.

Forced vibrations :-

When a repeated force continuously acts on a system, the vibrations are said to be forced.

Period :-

It is that the time taken by the motion to repeat it self and it is measure in seconds.

Cycle :-

It is the motion completed during the one period of time.

Resonance :-

When the frequency of external force is the same as that of the natural frequency of the system, a state of resonance is.

Said to have been reached * resonance results in large amplitudes of vibrations and this may be dangerous.

Types of vibrations:

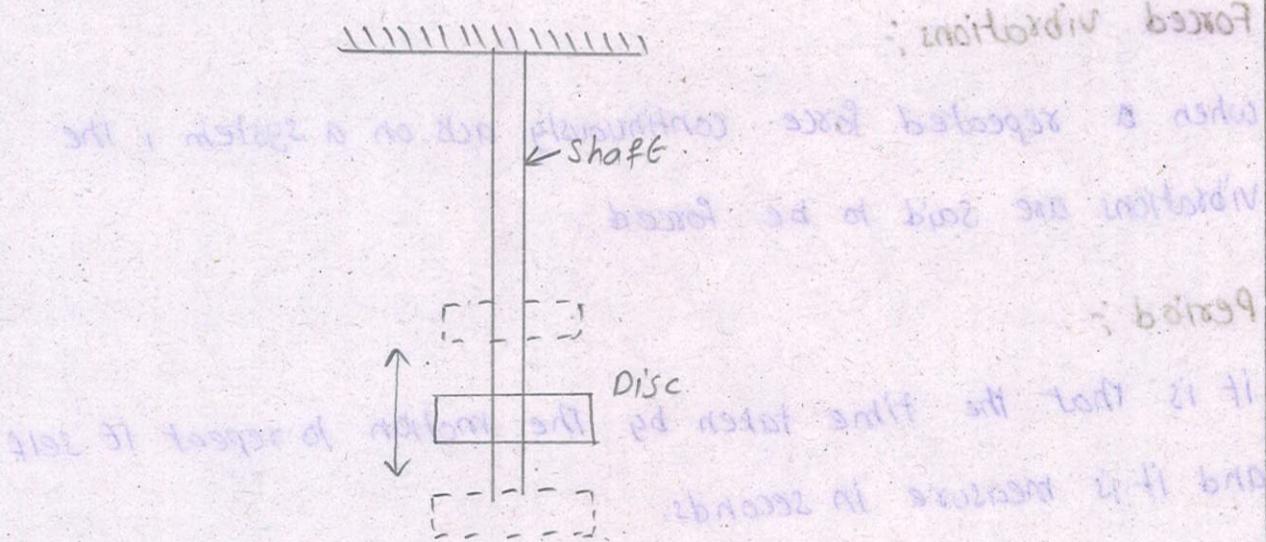
1) longitudinal vibrations

2) transverse vibrations

3) torsional vibrations

longitudinal vibrations:-

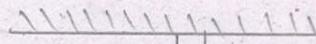
If the shaft is elongated and shortened so that the same moves up and down resulting in tensile and compressive stresses in the shaft, the vibrations are said to be longitudinal. The different particles of the body move parallel to the axis of the body.



Transverse vibrations:-

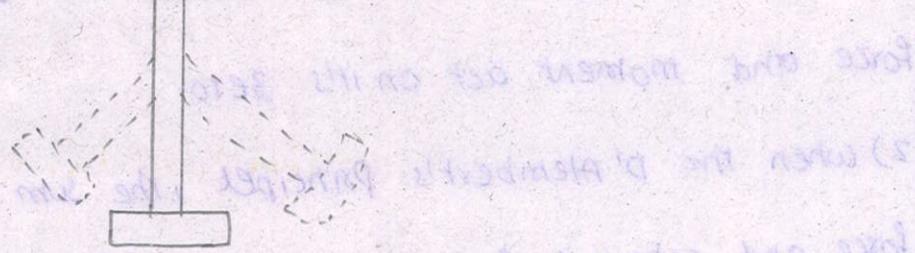
when the shaft is bent alternately and tensile and compressive stress due to bending results, the vibrations are said to be transverse. The particles of the body move approximately \perp to the axis of the shaft instead of perpendicular rotation.

inertia's moment about the axis



the moment of inertia about the longitudinal axis

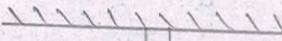
and stiffness of the longitudinal axis



(b)

Torsional vibrations :-

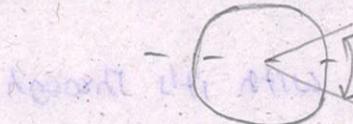
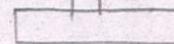
when the shaft is twisted and untwisted alternately and the torsional shear stresses are induced, the vibrations are known as torsional vibrations. The particles of the body move in a circle about the axis of the body movie circle about the axis of the shaft.



at each end no rotation occurs & at the center of the

axis there is no rotation

$$\theta = x^2 + \omega^2$$



$$\theta = x^2 + \omega^2$$

total angle made by the shaft

$$\theta_{\text{total}} = \theta_1 + \theta_2 = \omega t$$

Free longitudinal vibrations.

- 1) It is based on the principle of the when every a vibration system is in the equilibrium. The algebraic sum of force and moment act on it's zero.
- 2) When the D'Alembert's principle, the sum of the inertia force and external force on the body then the equilibrium must be "zero".

Let S = stiffness of the spring & weight of the mass ' m '

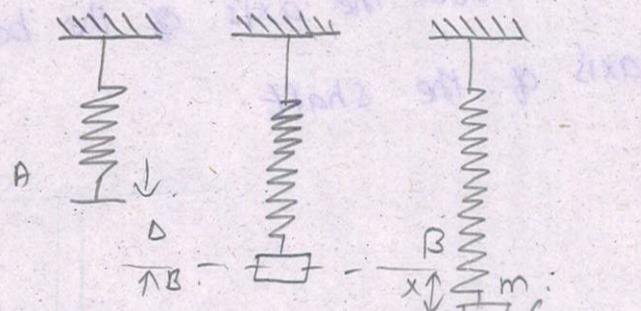
The static equilibrium position.

$$\therefore \text{upward force} = \text{downward force}$$

$$Sx_0 = mxg$$

$$\text{Inertia force} = m\ddot{x}$$

$$\text{Spring force} = Sx$$



\therefore sum of inertia force & external force on the body in any direction must be 0 .

$$m\ddot{x} + Sx = 0$$

$$\ddot{x} + \frac{S}{m}x = 0$$

This eqn of S.H.M along with its through.

$$\ddot{x} + \omega_n^2 x = 0$$

Then the soln given that

$$x = A \sin \omega_n t + B \cos \omega_n t$$

where $A \& B$ are constants. Their values depend upon the manner in which the vibration by assuming

$$A = x \cos \psi$$

$$B = x \sin \psi$$

where $x = x \cos \psi \sin \omega_n t + x \sin \psi \cos \omega_n t$

$$x = x [\sin(\omega_n t + \psi)]$$

$$\text{let assuming } A = x \sin \psi$$

$$B = x \cos \psi$$

$$x = x \sin \psi \sin \omega_n t + x \cos \psi \cos \omega_n t$$

$$x = x [\cos(\omega_n t - \psi)]$$

The above so that the system vibrates with the frequency,

$$\omega_n = \sqrt{\frac{s}{m}}$$

→ where is known as the natural circular frequency of vibrations.

→ As one cycle of motion is completed in the angle 2π .

The time period of vibration is $\frac{2\pi}{\omega_n}$

$$T = 2\pi \sqrt{\frac{m}{s}}$$

→ Natural linear frequency of vibrations system

$$F_n = \frac{1}{T}$$

$$F_n = \frac{1}{2\pi} \sqrt{\frac{s}{m}} = \frac{1}{2\pi} \sqrt{\frac{1}{(2\pi)^2 \cdot 0}} = \frac{1}{2\pi}$$

Dunkerly's method (for shafts having static deflections)

→ let $w_1, w_2, w_3, \dots, w_n$ if the concentrated loads on the shaft due to the mass $m_1, m_2, m_3, \dots, m_n$ & $D_1, D_2, D_3, \dots, D_n$ are the static deflections of this shafts under each load.

→ when that load act along on the shaft, if the shaft carries a uniform distributed mass of m unit length over it holes. Then δ is the static deflection at the midpoint of the load due to the mass be the delta deflections.

f_n^2 = Frequency of transfer vibration of the hole system.

f_{ns}^2 = Frequency in the distribution load acting along the body.

Then the acco to the Dunkerly's empirical formula is.

$$\frac{1}{f_n^2} = \frac{1}{f_{n1}^2} + \frac{1}{f_{n2}^2} + \frac{1}{f_{n3}^2} + \dots + \frac{1}{f_{ns}^2}$$

$$\text{where } \frac{1}{f_i^2} = \frac{1}{2\pi} \sqrt{\frac{g}{D}} = \frac{1}{2\pi} \times \sqrt{\frac{9.81}{D_i}} = \frac{0.4985}{\sqrt{D_i}}$$

$$\frac{1}{f_{ns}^2} = \frac{\pi}{2} \sqrt{\frac{59}{384\Delta}} = \frac{\pi}{2\pi} \times \sqrt{\frac{59 \times 9.81}{384 \times \Delta_{ns}}} = \frac{0.5614}{\sqrt{\Delta_{ns}}}$$

$$\text{let } \frac{1}{f_n^2} = \frac{1}{(0.4985)^2} + \frac{1}{(\frac{0.4985}{\sqrt{D_1}})^2} + \dots + \frac{1}{(\frac{0.5614}{\sqrt{D_s}})^2}$$

$$\frac{1}{f_n^2} = \frac{D_1}{(0.4985)^2} + \frac{D_2}{(0.4985)^2} + \frac{D_3}{(0.4985)^2} + \dots + \frac{D_s}{(0.5614)^2}$$

$$\frac{1}{f_n^2} = \frac{1}{(0.4985)^2} (D_1 + D_2 + D_3) + \dots + \frac{D_s}{(0.5614)^2}$$

$$f_n = \frac{0.4985}{\sqrt{D_1 + D_2 + D_3 + \dots + D_s}}$$

1.27.

$$\times 2 \times \frac{1}{f} = (\omega n)^2 \times \frac{1}{f}$$

$$\times 2 < \omega^2 \times m$$

$$\frac{1}{m} = \omega^2$$

$$\frac{1}{m} = \omega^2$$

Raleigh's method.

→ in this method the max K.E and the vane position is made equal to the max potential at the extreme position.

Let S.H.M $x = x \sin \omega_n t$

$$x = x \cos \omega_n t$$

$$x = x \sin \omega_n t$$

1.0.bj.

$$x = x \cos \omega_n t \omega_n$$

$$x = x \omega_n \cos \omega_n t$$

$$\text{let } \theta = t$$

$$t = 0$$

$$\dot{x} = x \omega_n$$

$$SI = \text{second}$$

$$DP = \text{displacement}$$

Let $\theta = t$ let \dot{x} called as drag. Do go mul 3rd till

$$t = 90^\circ$$

abcd. sti no point except another = DP - 30/30

$$\boxed{\dot{x} = 0}$$

$$= AP + BI$$

→ let us consider the K.E of mean position is equal to the P.E of extrem position.

$$K.E = P.E.$$

$$\frac{1}{2} m \dot{x}^2 = \frac{1}{2} I \omega_n^2 \theta$$

$$m \omega_n^2 = \frac{I}{2}$$

$$\frac{I}{T} \cdot \frac{1}{BS} = \frac{I}{2}$$

$$\frac{1}{2}mv^2 = \frac{1}{L}m\Delta^2 + \dots + (\epsilon A + A + \alpha) \quad \frac{1}{(2\pi f \cdot 0)} = \frac{1}{\omega_n^2}$$

$$\frac{1}{2}m(x\omega_n)^2 = \frac{1}{L}s \cdot x^2.$$

$$m x^2 \omega_n^2 = s x^2$$

$$\omega_n^2 = \frac{s}{m}$$

$$\omega_n = \sqrt{\frac{s}{m}}$$

bordom undies

Free torsional vibrations. At both ends fixed.

i) single rotor torsional free vibrations.

→ let us consider a uniform shaft of length L : fix at a upper end and carrying disc of moment of inertia I . The shaft is assumed mass less. A disc is given its twist about its vertical axis and the release. It starts that an oscillating about the axis and will perform torsional vibration.

→ At any instant of time acting on the disc

i) inertia force $= -I\ddot{\theta}$

ii) restoring torque $= q\theta$

iii) The sum of all parts are acting on the disc must be 0 :

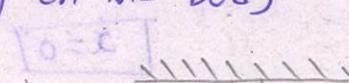
where $-q\theta =$ resistance torque acting on the body.

$$I\ddot{\theta} + q\dot{\theta} = 0$$

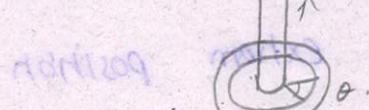
$\ddot{\theta} + \left(\frac{q}{I}\right)\dot{\theta} = 0$ (from $\ddot{\theta} + \alpha\dot{\theta} + \epsilon\theta = 0$)

where $\frac{q}{I} = \omega_n$.

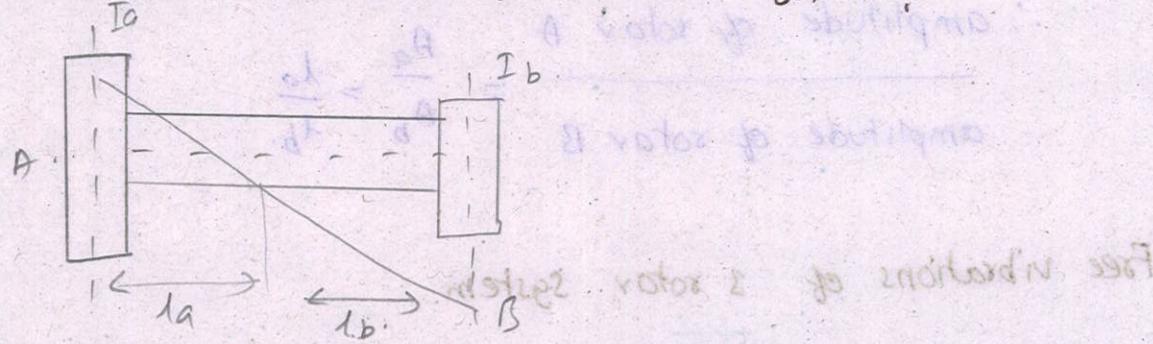
$$f_n = \frac{1}{2\pi} \sqrt{\frac{q}{I}} \quad \text{where } q = \frac{GJ}{L}$$



due to rotation θ about the axis.



Free torsional vibrations of 2 rotor system:



→ If the shaft held in the bearing carries at rotor each end it can vibrate torsionally such that 2 rotors move in the opposite direction. Thus, so length of the shaft is twisted in one direction & in the rest in the twisted in other. The section which does not undergo any twist is called nodal section.

→ When the frequencies of A & B acting equal position in the shaft,

$$F_{na} = F_{nb}$$

$$\frac{1}{2\pi} \sqrt{\frac{\tau_a}{I_a}} = \frac{1}{2\pi} \sqrt{\frac{\tau_b}{I_b}}$$

$$\text{where } \tau = \frac{GJ}{l}$$

$$\frac{GJ}{I_a l_a} = \frac{GJ}{I_b l_b}$$

$$\frac{I_a}{I_b} = \frac{l_b}{l_a}$$

$$\frac{1}{2\pi} \sqrt{\frac{1}{l_a}} = \frac{1}{2\pi} \sqrt{\frac{1}{l_b}} = \frac{1}{2\pi} \sqrt{\frac{1}{l}}$$

$$\frac{1}{l_a} = \frac{1}{l_b} = \frac{1}{l}$$

$$\frac{1}{l_a} = \frac{1}{l_b} = \frac{1}{l}$$

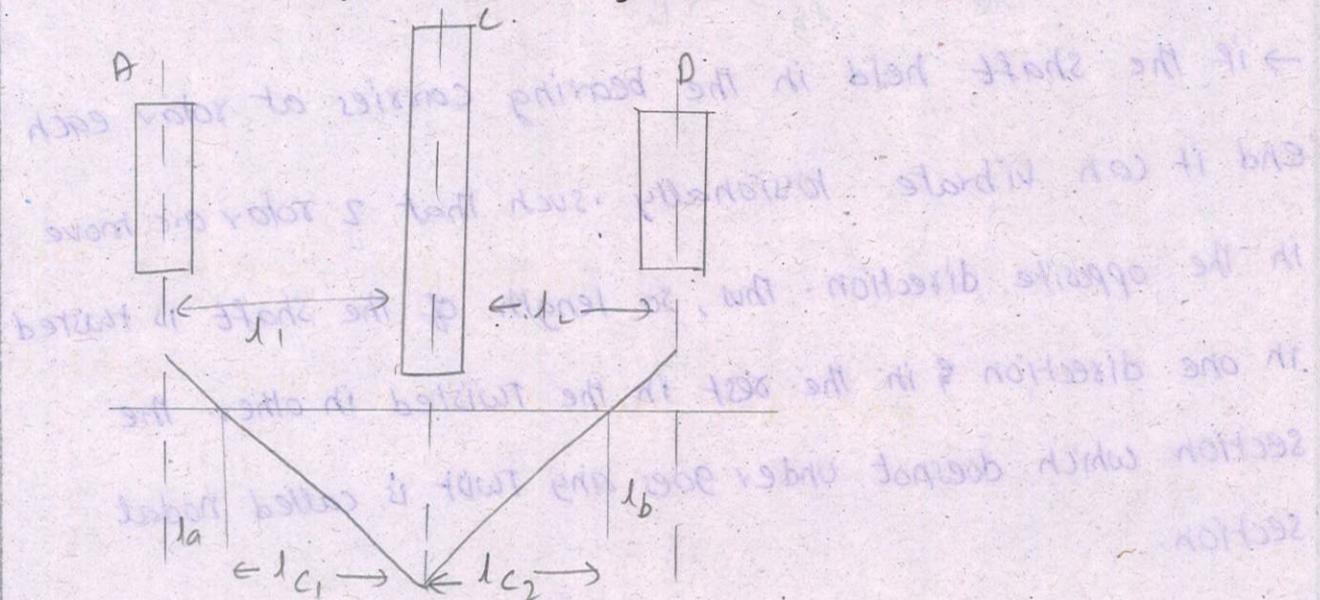
→ Thus the nodes divides the length of the shaft in the inverse ratio of MoI of the 2 rotors.

$$\frac{1}{l_{total}} = \frac{1}{l_a} \left(\frac{1}{l_a} + \frac{1}{l_b} \right) = \frac{1}{l_a l_b}$$

\therefore amplitudes of rotors A & B are in direct proportion to their moments of inertia.

$$\frac{\text{amplitude of rotor } A}{\text{amplitude of rotor } B} = \frac{I_a}{I_b} = \frac{A_a}{A_b} = \frac{l_a}{l_b}$$

Free vibrations of 3 rotor system.



→ Consider a 3 rotor system in which the rotors A & B are fixed at the end of the shaft & rotor 'C' is free. The rotors A & B rotate in same directions & 'C' is in opposite direction and nodes are acting D & E.

$$F_{na} = f_{nc} = f_{nb}$$

$$\frac{1}{2\pi} \sqrt{\frac{g_a}{I_a}} = \frac{1}{2\pi} \sqrt{\frac{g_c}{I_c}} = \frac{1}{2\pi} \sqrt{\frac{g_b}{I_b}}$$

$$\frac{g_a}{I_a} = \frac{g_c}{I_c} = \frac{g_b}{I_b}$$

$$\frac{g_a}{I_a I_a} = \frac{g_c}{I_c I_c} = \frac{g_b}{I_b I_b}$$

$$\frac{1}{I_a I_a} = \frac{1}{I_c I_c} = \frac{1}{I_b I_b}$$

$$\Rightarrow \frac{1}{I_a I_a} = \left(\frac{1}{l_1 - l_a} + \frac{1}{l_2 - l_b} \right) \frac{1}{I_c} = \frac{1}{I_b I_b}$$